Bitcoin as an investment asset:

The added value of bitcoin in a global market portfolio

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Masterthesis

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Abstract

This paper investigates whether bitcoin as an investment asset offers diversification benefits, more specific whether bitcoin has hedge or safe haven properties. Bitcoin is a decentralized digital cryptographic currency which has gone through and still goes through spectacular developments leading to a lot of media but also academic attention. Recent literature shows that bitcoin can be seen more as an investment asset than a currency and that bitcoin is useful both for risk management as portfolio optimization. This paper uses the mean variance framework, which can uniquely incorporate policy constraints, in combination with the Monte Carlo Simulation to address the estimation risk issue which is considers as an important aspect for a very volatile asset such as bitcoin. Approaching the performance of bitcoin from an global investment point of view puts bitcoin into a new perspective. The findings of the paper are consistent and show that bitcoin is an effective diversifier with on average a weight allocation between 0% to 5%. Bitcoin shows no hedge or safe haven properties for a global market portfolio. Even though bitcoin shows very robust results investing in bitcoin comes with certain risk which are inherent to bitcoin characteristics and bitcoin's usage.

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1 Introduction

One of the big questions in finance is whether to buy a certain asset, be it a stock a bond or another financial instrument, and add the asset to an investment portfolio. This paper will zoom in on the behavior of a relatively new asset, namely bitcoin, in a global market portfolio. Optimally an investment portfolio would generate high returns for a low as possible risk, in good but also in bad times. Harry Markowitz (1952) build the foundations for portfolio optimization. Markowitz was the first to introduce 'diversification' into an economic theory which determines how an investor maximizes his portfolio choice. Markowitz rejected the rule that an investor maximizes the discounted value of future returns because no matter how the discount rates and future value varies the rule fails to imply diversification.

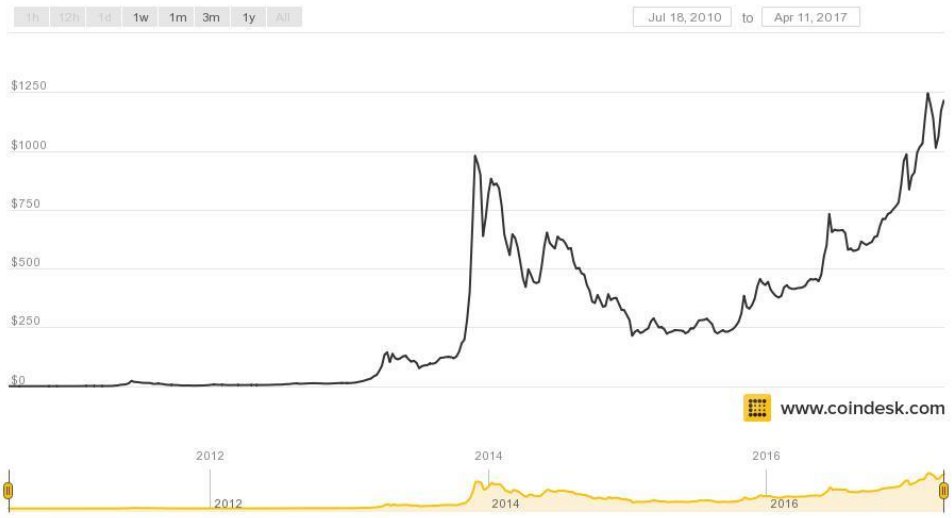
Diversification is one of the key elements this paper focuses on. Especially during a financial crisis because stocks appear to be highly correlated in such times. Moldovan (2011) looks upon the stock correlations of the three greatest financial centers in the world (New York, London and Tokyo) before and during the recent financial crisis. Moldovan shows with multiple regressions that the link between the three stock markets were more intense during the crisis. Numerous other papers, with different methods and regions, find similar results that during financial crisis stock markets become higher correlated (Sandaval Junior and De Paula Franca, 2012; Kenourgious et al., 2011; Sylljgnakis and Kouretas 2011). According to Silvennoinen and Thorp (2011) do not only the correlations between stock markets increase but also between stock and bond markets and even stock, bond and commodity markets. Especially during the period 2007-2009. Forbes and Rigobon (2002) confirm the high correlation even though that is not their primary goal. Forbes and Rigobon test whether there is contagion or only interdependence across stock markets. Although they do not find evidence in favor of contagion, meaning that the linkage between markets significantly increase, they do find a continuous high correlation between the markets.

The high correlations between stocks markets during bad times are troublesome for investors because it undermines the diversification principle. Luckily asset classified as hedges or safe havens are there to save the day. The general idea is that these assets in bad times perform well. The economic literature is quite rich on the concepts of hedges and safe havens, but only on traditional assets such as: precious metals, currencies and bonds. Where a lot of attention is paid to Gold and the US dollar. According to Baur and McDermontt (2010) is gold a safe haven. Further they show that gold is not only a safe haven for crisis periods but also for periods with increased uncertainty. Baur and Lucey (2010) confirm that gold is a safe

haven for stocks. Baur and Lucey also found that gold is no safe haven for bonds. Beckmann et al. (2014) confirm the hypotheses of Baur and Lucey (2010), gold is a hedge and a safe haven. Hillier et al. (2006) examine whether Gold, Platinum and Silver offer diversification benefits because of their low correlations with stocks. The finding of the paper is that there are diversification benefits but more important is that all three precious metals show hedge capabilities. However, according to Bouoiyour and Selmi (2017) have precious metals lost their safe haven properties over time. Kaul and Sapp (2006) and Diekmann and Meurers (2007) both acknowledge that the US dollar is a safe haven. Ranaldo and Söderlind (2010) show that the Swiss franc but also the yen and the euro behave as a safe haven currency in distressed times. At the same time they show that the US dollar is pro-cyclical with equity markets, the opposite of the previous findings. Grisse and Nitschka (2013) find that the Swiss franc is a safe haven currency while Botman et al. 2013 find that the yen is a safe haven currency in a particular case.

Even though the Literature is quite vast on hedge and safe haven assets a relative new and unique asset has almost not been studied before. Bitcoin a new decentralized digital currency originating from 2008 has grown remarkably in size since its birth, from 0.07 cents to a high of 1244.61, as is visible in the figure below. With a market capitalization of 19.77 billion dollar bitcoin becomes a serious player in the market.

Figure 1: Market capitalization of Bitcoin.



Source: www.coindesk.com

The growth of the market capitalization of bitcoin goes in parallel with the growth of the media attention but also with a growing interest in the economic literature. Brandvold et al. (2015) and Ciaian et al. (2016) examine the price mechanism of the bitcoin market. Rogojanu

and Badea (2014) compare bitcoin to alternative monetary systems but conclude that "*Bitcoin is like gold, but in a virtual environment*". The same comparison between bitcoin and gold is made in Popper (2015) and Dyhrberg (2015a). Dyhrberg (2015b) compares bitcoin not only to gold but also to the US dollar. Yermack (2013) argues that bitcoin is a speculative investment instead of a true currency. Because of the comparison of bitcoin to gold but also the US dollar the question arises whether bitcoin is useful investment purposes. Eisl et al. (2015) and Halaburda and Gandal (2014) shows that bitcoin is a profitable investment.

The spectacular development of bitcoin in the past year led to speculations in the media about bitcoin being the modern hedge and or safe haven. The Dutch media mentioned that bitcoin is a hedge for stocks during the Brexit and the US elections of 2016 (FD, 2016). Shaffer (2017) from CNBC speculates about bitcoin rivalling gold as a safe haven while Ford (2013) from Bloomberg speculates about bitcoin being the last safe haven. Recent studies show that bitcoin can show signs of a hedge and safe haven. Bouri et al. (2017) show that bitcoin is a poor hedge and is only suitable for diversifying purposes. Only in the case of weekly extreme down movements is bitcoin a strong safe haven. Dyhrberg (2015a) find that bitcoin is a hedge against the FTSE and that in the short term a hedge against the US Dollar. Dyhrberg (2015b) finds that bitcoin can be useful in risk management when a negative shock is expected. According to Bouoiyour and Selmi (2017) is bitcoin safe haven property time-varying and has primarily been a weak safe haven in the short and long term.

Previous findings of bitcoin as a hedge or safe haven are inconsistent and lack a fundamental base. This paper will attempt to answer two questions which has not been in answered in an adequate matter yet. The first question is:

- *Does bitcoin offer diversification advantages for a global market portfolio?*

The second question is an extension of the first and is:

- *Is bitcoin a hedge or a safe haven for a global market portfolio.*

This paper distinguishes from the recent literature through three main channels. First, where previous papers look at specific market segments this paper will approach the question from a global investor perspective. Second, bitcoin has several unique characteristics which are important to take in to consideration before investing in bitcoin. Other papers so far do not take these considerations adequately into account. Third, previous literature used different methods such as GARCH models while this paper uses the mean variance framework in combination with the Monte Carlo Simulation giving it several advantages.

The mean variance framework can uniquely incorporate policy constraint, such as no short sales, by altering the weight function. Section 2 and 3 will give a better understanding about this. Instead of the traditional mean variance model a simulated mean variance model will be applied. The traditional model does not take estimation risk in to account. Estimation risk is defined as followed: "*The possibility of errors in the portfolio allocations due to imprecision in the estimated inputs to the portfolio optimization*" (Jorion, 1992, p. 70). Due to the high volatility of bitcoin (Molnár et al., 2015; Baur and Dimpfl, 2017) the estimation error is considered as a very important factor. As a consequence, the results of the traditional mean variance model will not be reliable. A solution to this problem is the Monte Carlos Simulation (MCS). Based on the historical values new data will be generated based on the model:

- $R_t = \mu + \varepsilon_t$.

The global market portfolio will consist of: several stock indexes (S&P 500, FTSE, DAX, Nikkei, Shanghai A share, MSCI world), several bonds (American, European, Asian), a commodity index and a real estate index. The return, the variance and the covariance matrix of the portfolio will be calculated along with the weights of each asset in the portfolio to construct the efficient frontier. Then bitcoin will be added to the portfolio to see whether it offers any advantages in terms of risk and return and what percentage has to be invested in bitcoin. Then the simulated data will be used to draw a random sample from the distribution and determine the market portfolio. Repeating the simulation one hundred times will result in a scatterplot of portfolios. By adding bitcoin and repeating the same process can be determined which portfolio performs better. The dataset consists of the period 2010-2016 with weekly data of all the assets and can easily be accessed via Eikon. The time period captures different economic cycles period making it a great opportunity to check whether bitcoin is a possible hedge or safe haven. In order to make the results more robust the market portfolio can be changed by adding or removing certain assets and by changing the periods slightly and by adopting different strategies.

The remainder of the paper will be organized as followed: Section 2 discusses the theoretical framework. Starting with an elaboration on bitcoin in order to clarify what bitcoin exactly is, how it functions and what the risks are when investing in bitcoin. Then portfolio optimization theory will briefly be discussed followed by a specific paragraph on the safe haven or hedge effect of bitcoin since that is the focus of the paper. Section 2 concludes with the hypotheses. Section 3 will present the data and will elaborate on the method of the

research. Section 4 will present the main results of the paper followed by section 5 which presents the robustness checks. The paper concludes by means of section 6.

2 Theoretical overview

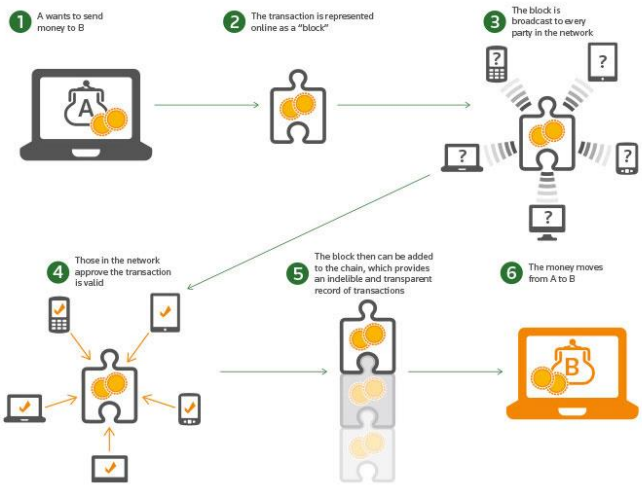
2.1 Bitcoin, what is it?

Before going in more detail about portfolio optimization theory and the hedge and safe haven concept one crucial aspect will be examined in detail, what is bitcoin? This question is justified for several reasons. First of all, the (economic) literature on bitcoin is quite thin. Second bitcoin itself is quite a complex investment asset as it has some unique characteristics. Third, with the unique characteristics come unique risks. Last, a unique characteristic which needs to be emphasized is that bitcoin is a currency with no monetary policy.

Bitcoin in a nutshell

bitcoin is a decentralized peer to peer digital cryptographic currency. The creators of bitcoin are unknown although Satoshi Nakamoto is often named as one of the developers of bitcoin (Barber et al. 2012). The bitcoin network relies on what is called a 'block chain technology'[1]. Blockchain technology works as followed (see figure 2 for a graphic explanation): A blockchain is basically a ledger which records all historical transactions[2][3][4], point 5 in figure 2. The ledger is controlled peer to peer, meaning that there are several nodes (computers) which are connected, point 3 in figure 2. Each node verifies a new action, point 4. Each action is considered as a block, point 2. Since the ledger is a historical overview of each block (transaction), it basically becomes a chain of blocks.

Figure 2: Blockchain technology



Source: https://blogs.thomsonreuters.com/answerson/blockchain-technology/

[1] https://bitcoin.org/en/how-it-works
[2] http://www.ibm.com/blockchain/what-is-blockchain.html
[3] http://www.pwc.com/us/en/technology-forecast/blockchain/definition.html
[4] http://www.blockchaintechnologies.com/blockchain-definition

Bitcoin's unique characteristics

Bitcoin has several unique characteristics. First there is no intervention from traditional financial institutions such as banks necessary (Nakamoto 2008). Because of the blockchain technology and the functions of the nodes no traditional third party such as a bank or a clearance house is needed. Also, implying that transactions can be done anywhere and at any time creating business flexibility. At the same time this system guarantees anonymity in the system. Second, there is no central authority or issuer (Reid and Harrigan, 2012). Bitcoins are generated through a 'mining' process (Rogojanu and Badea, 2014) which results in a predictable growth rate, which is a third unique characteristic. The second and third characteristics are especially interesting because this means that no government has any influence on the currency. No monetary policy can appreciate/ depreciate or even revaluate/devaluate the currency. Which takes some uncertainty away and makes the currency less likely to be a target of speculation. Fourth, there is no trust issue of financial institutions because of the usage of cryptography (Barber et al. 2012). Each node verifies each transaction in the ledger, therefore no institution intervenes and no trust issues arise. Bitcoin itself does not generate inflation. Limiting the money supply of a currency can be seen as a great advantage in fighting inflation, the money supply of bitcoin is fixed and predictable and the total number of bitcoins in the end is known (21 million) (Burghelea, 2008; Ciaian et al., 2015).

The risks of bitcoin

There are two main risk components of bitcoin which need further elaboration because most assets do not experience these risks. First, as is already mentioned in the introduction is that bitcoins price is extremely volatile (Molnár, 2015; Baur and Dimpfl, 2017). The extreme volatility is an extra dimension when considering to invest in bitcoin. From a risk neutral perspective it does not matter but consider the loss aversion concept of Kahneman and Tversky (1979) which shows that losses weigh more than gains. An asset with larger volatility will generate less utility, which practically would be an important consideration for investors. A second issue is that bitcoin, unlike traditional currencies, is susceptible for hackers. By means of malware attacks or cyber-attacks there are possibilities to steal bitcoin and to destabilize the system which can generate even higher price volatility (Barber et al., 2012; Ciaian et al., 2016). According to Moore and Christin (2013), who examined 40 Bitcoin exchanges, have 18 of the 40 exchanges been closed due to cyber-attacks. In 2014 even the

largest bitcoin exchange collapsed because of a cyber-attack. Since bitcoin is and will be a digital currency this will be a continuous threat and sometimes a reality.

Bitcoin is unique because of several characteristics. Even though some of these characteristics give great advantage to bitcoin some also cause issues. Two main issues which hinder the working of bitcoin will be addressed shortly.

The first issue is that bitcoin is a digital currency. A currency can have three functions: a medium of exchange, a unit of account and a store of value. The medium of exchange is often seen as the most important function (Mishkin, 2009). Bitcoin has problems with the first two functions. According to Yermack (2013) is bitcoin a speculative investment instead of a currency. Bitcoins consumer transaction volume is very low. The volatility is much higher compared to widely used currencies, creating a large short-term risk for the scare users. Bitcoin is thus not functioning well as a medium of exchange. Bitcoin prices of consumer goods require very odd numbers with leading zeros which is disturbing for retail market participants, making bitcoin not functioning well as a unit of account. Baur et al. (2015) confirm that bitcoin is more an investment asset than a currency. Over one third of the users only use the currency for investment purposes. Bitcoin does apparently function well as a store of value.

The second issue is that according to Rogojanu and Badea (2014) bitcoin is popular among gamblers and that bitcoin is used in illegal activities such as: tax evasion, terrorism and facilitating transactions of goods prohibited by law (mainly drugs and weapons). Agencies such as the FBI and Europol are actively fighting against internet organized crime. The FBI closed the Silk Road underground marketplace which sold drugs and narcotics using bitcoin as only transaction possibility (Moore, 2013). The Internet Organized Crime Threat Assessment (IOCTA) report of 2015 by Europol gives more insight about the seriousness of internet crime using bitcoin. "*Bitcoin is establishing itself as a single common currency for cybercriminals within the EU.*" (IOCTA, 2015, p. 11) and "*Overall, Bitcoin is beginning to feature heavily in many EU law enforcement investigations, accounting for over 40% of all identified criminal-to-criminal payments.*" (IOCTA, 2015, p. 46). The usage of bitcoin in the criminal circuit leads to question marks whether bitcoin should be banned or not. Thailand banned bitcoin in 2013 (Trotman, 2013) and since the beginning of this year China is warning investors for a possible ban in China which heavenly influences the price of bitcoin (Dai and Lee, 2017)

The case for bitcoin as an investment asset in a portfolio

As Yermack (2013) and Baur (2015) state bitcoin is more an investment asset than a currency. Adopting bitcoin in a global market portfolio could be interesting from the investment perspective. The reason why this could be interesting is that bitcoin in the long run is unrelated to global macroeconomics and financial developments. Bitcoin instead is only sensitive to two forces; one is the bitcoin market forces of supply/demand and the other one is digital-currency-specific factors such as attractiveness of bitcoin (Ciaian et al., 2016). In the next paragraph will be explained why exposure to different factors can be beneficial and in paragraph three will the results from previous studies on this matter be presented.

2.2 Portfolio optimization

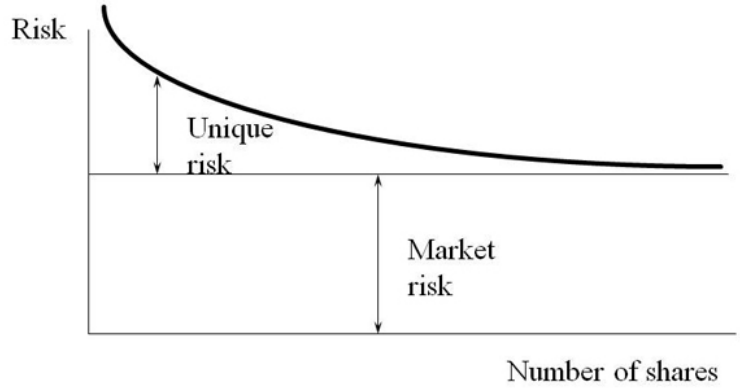
Previous paragraph gave an understanding about what bitcoin is, how bitcoin works and the unique features bitcoin has in order to give a better understanding about bitcoin as an investment assets. This paragraph will look at the fundamentals of portfolio optimization and explain how bitcoin could potentially be very useful for global market investors.

"*Diversification cannot eliminate all variance"* (Markowitz, 1952, p.79). The risk-reward ratio is an important ratio for each investor. Important is that there are two main risk factors which investors are concerned with. That is market risk and firm-specific risk.

Firm-specific risk also known as non-systematic risk, unique risk or diversifiable risk. Unique risk is determined by micro-economic factors. Each micro-economic factor only influences the specific firm. Well-known examples of firm-specific risks are the Small minus Big and High minus Low from Fama and French (1993). Be means of diversification can the firm-specific risk be eliminated. Which is visible in figure 3, where firm specific is the convex line showing that the higher the number of assets the lower the firm-specific risk (Bodie, Kane and Markus, 2014). However, this is not entirely true. This will be explained after the market risk.

Market risk also known as systematic risk or non-diversifiable risk. Characteristics for market risk is that it is inherent to the investment in the market. By no means can this form of risk be lowered by any amount. Market risk is exposed to macro-economic factors such as conjuncture cycles and interest rates. Each factor influences the market as a whole (Bodie, Kane and Markus, 2014). In figure 3 market risk is indicated by the horizontal straight line. No matter the number of assets the market risk will always be the same. In other words, diversification cannot eliminate all risk.

Figure 2: unique risk and market risk



Source: Bodie, Kane and Markus, (2014)

A side-note on the risk part is that there is a third form of risk, namely industrial risk. This form of risk can be placed between market risk and firm-specific risk. Industrial risk only influences a certain industry. For example, when there is a really dry year with lots of sun, this might be bad for the agriculture sector but at the same time this is great for the energy sector which specifies on solar panels. Other firms will likely not be affected by the unpredictable weather. This implies that just investing in more assets does not mean that all firm-specific risk can be eliminated (Manning and Napier, 2015). Which Markowitz (1952, p. 89) already mentioned: "the *"right kind" of diversification for the "right reason."* "

The principle of the right kind of diversification for the right reason is based on the following: Assume there is a portfolio with two risky assets, Asset X and Asset Y. With expected return r1 and r2 and variance of σ_1 and σ_2. The expected return and variance is as followed:

(1) $E(rp) = w_1 \times E(r1) + w_2 \times E(r2)$

(2) $var(rp) = w_1^2\, var\,(r1) + w_2^2\, var(r2) + 2w_1\, w_2\, cov(r1,r2)$

Where w denotes the weight imposed on the asset. The variance can be noted as σ^2. The covariance is the correlation between the asset 1 and 2, denoted as ρ, and their variance.

(3) $\rho = \frac{cov(r1,r2)}{\sqrt{var(r1)var(r2)}} = \frac{cov\,(r1,r2)}{\sigma 1\, \sigma 2}$

Rewriting formula 3 and substituting this in formula 2 gives:

(4) $\sigma_p^2 = w_1^2\, \sigma_1^2 + w_2^2\, \sigma_2^2 + 2\, w_1\, w_2\, \rho_{12}\, \sigma_1\, \sigma_2$

When there is a perfect positive correlation ρ_{12} is equal to 1. There is a linear trade-off between the two risk assets in return and variance because the variance is as followed:

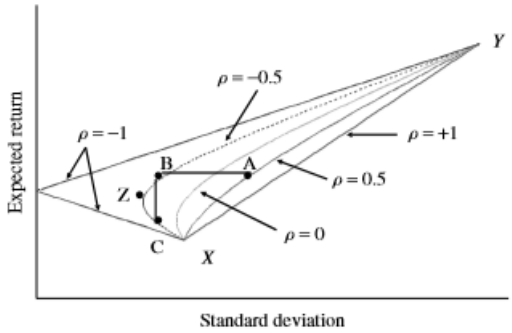
(5) $\sigma_p^2 = [w_1 \sigma_1 + (1-w_1)\sigma_2]^2$

When ρ_{12} is equal to -1 there is a perfect negative correlation and theoretically and risk free return can be realized because the variance becomes as followed:

(6) $\sigma_p^2 = [w_1 \sigma_1 + (1-w_1)\sigma_2]^2$

Note, in formula 5 and 6 w_2 is replaced by $1-w_1$. Since the investment consists of two assets the total weights should be 1, therefore is $1-w_1$ equal to w_2.
Combining formula 1 and 4 into a graph where gives the following:

Figure 4: Efficient frontier and correlations



Source: Cuthbertson and Nitzsche (2004)

Figure 4 shows how the portfolio of the two-risky asset varies when the correlation between the two assets varies. The lower the correlation between the two assets the higher the return-risk ratio. An important aspect is the mean-variance dominance criteria, which implies that point B is preferred to point C because the standard deviation is the same only the return of B is higher. Furthermore, B is preferred to A since both have the same return but the standard deviation of B is lower than A. In short, the upperpart (the concave line from Z upwards) is the line with the efficient portfolios. This part is also known as the efficient frontier.

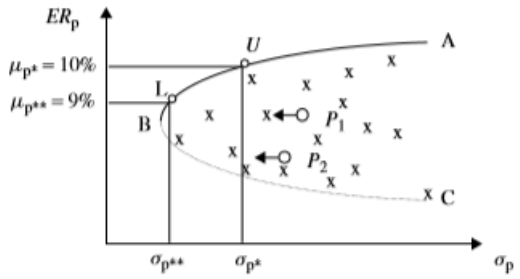
The two-asset portfolio framework can be extended to a N assets portfolio. Where the weights are w_i (i = 1, 2,..., N) Assume that n expected returns μ_i, variance σ_i^2 and n(n-1)/2 covariances σ_{ij} (or as formula 3 showed the correlation coefficients ρ_{ij}) are known. The formula for the return and the variance of the portfolio are as followed:

(7) $$\mu_p = \sum_{i=1}^{n} w_i \mu_i$$

(8) $$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

For an investor assume that he is only concerned about return and risk when determining all w_i and that the investor's budget constraint is $\sum w_i = 1$. In this model are short sales permitted ($w_i < 0$). The following figure can be drawn.

Figure 5: Efficient frontier N assets



Source: Cuthbertson and Nitzsche (2004)

Figure 5 clearly shows the range of portfolios and their efficiency. Every Portfolio on the line B-A, the efficient frontier, should be preferred to portfolios on line B-C because of the previous mentioned mean-variance dominance criteria. (For more information about the separation principle and the CML see Cuthberston and Nitzsche (2004) chapter 5.)

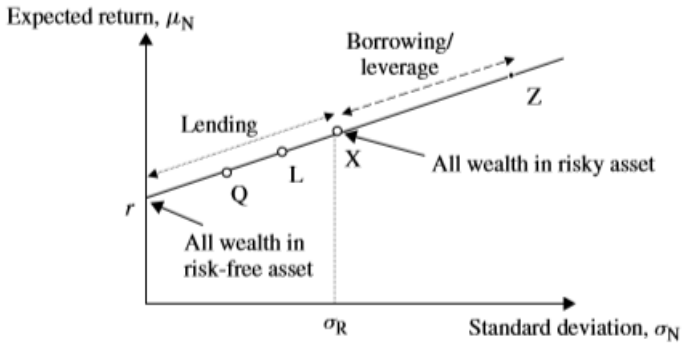
Even though this model shows nicely how the efficient frontier can be constructed two important question about portfolio optimization arise: How many asset are needed in order to diversify away firm-specific risk? and is there an *efficient* way of diversification? To start with the first one, several studies conducted research on the matter and the results differ quite a lot. Evans and Archer (1968) report that with already 10 different stocks the optimal portfolio can be achieved in the sense that all firm-specific risk is gone and the portfolio is only exposed to market risk. Statman (1987) reports that almost 40 different stocks are necessary in order to diversify away the firm-specific risk. Other studies such as Malkiel (1999), Burnside (2004) and Graham (2009) document findings between Evans and Archer (1969) and Statman (1987). On average 25 different stocks suffices. All papers show similar figures as figure 2. Furthermore, the advantage of the diversification principle strongly decreases after about 8 different stocks.

As stated before, increasing the number of stocks in a portfolio can decrease the risk of a portfolio. An important remark is that this does not necessarily lead to an optimal portfolio.

For an optimal portfolio, *efficient* diversification is very important. Efficient diversification can be achieved through investments in low correlated assets or even negative correlated assets (Bodie, Kane and Markus, 2014). The lower the correlation, ρ_{ij} in formula 8, the more efficient the diversification effect is. The most important contribution of Malkiel (1999) is that the number of shares is not that important but that the correlation between asset is very important in achieving a well-diversified portfolio. Coming back to the side note about industrial risk, because all companies in the same industries are partly affected by the same factors their correlation is relatively high. Increasing the number of shares of the same portfolio does not have to change the portfolio's risk.

The efficient frontier only shows given a certain return what the minimum variance portfolio is. The question remains what portfolio is the best or in other words the market portfolio? To answer this question let's first introduce the transformation line. The transformation line represents all points where an investor can invest in two assets. One asset is the risk-free rate and the other asset is a stock portfolio. The investor can invest in any combination between the two assets. By lending at the risk free rate the investor can invest more than his initial wealth in the portfolio. Figure 6 shows an example of a transformation line.

Figure 6: the transformation line

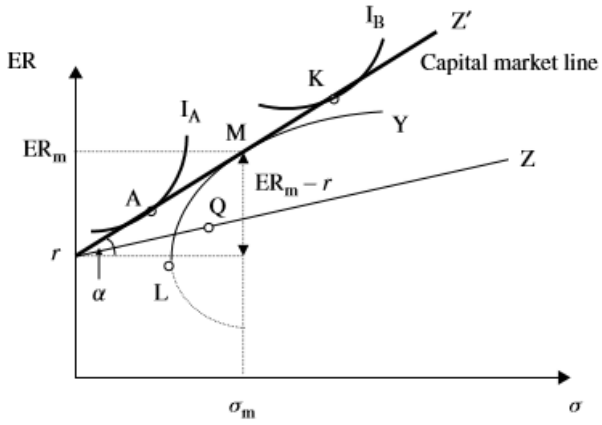


Source: Cuthbertson and Nitzsche (2004)

The transformation line gives a linear risk return relationship between the two assets. The relationship is described as $\frac{Rp-Rf}{\sigma}$. A transformation line with a steeper slope means a higher risk return reward. When the transformation line and the efficient frontier are combined an efficient portfolio with the highest risk-return ratio can be selected. This

portfolio is the market portfolio[5], see figure 7. The highest transformation line which is tangent to the efficient frontier is called the capital market line (CML). Each investor should invest in the market portfolio regardless of investor preferences. The preferences only play a role where to invest along the line rZ', as is indicated by the two different preference curves I_a and I_b. The process of first determining the market portfolio and the weights of each individual asset in the market portfolio followed by determining which proportion of the total portfolio will be invested in the market portfolio is known as the separation principle.

Figure 7: Portfolio choice



Source: Cuthbertson and Nitzsche (2004)

Correlation values are thus key for portfolio optimization. This is where Hedges and safe havens come in to play. By definition of both concepts are the correlation values negative for either the whole period or only in times of distress (for the exact definition see the next paragraph). As mentioned in the introduction is that Forbes and Rigobon (2002) show that during times of turmoil stocks markets correlations increase. This increase in stock market correlations lowers the efficient diversification of the portfolio. However, as just mentioned is that hedges and safe havens are negatively correlated. Combining both, high stock correlations and negative hedge/safe haven correlations could be a very interesting way to optimize portfolio.

[5] A critical note on the determination of the market portfolio is placed by Roll (1977). Roll states that for each test of the market portfolio the presumption is that there is complete knowledge of the composition of the 'true' market portfolio. Implying that every individual asset must be included in a correct test, which is not feasible. This also implies that there is no more room for diversification since all assets are already taken into account.

2.3 Bitcoin's properties

The main focus of the paper is to discover whether Bitcoin offers diversification advantages, more specific is whether Bitcoin is a hedge or safe haven. Throughout the literature several definitions are applied to hedges and safe havens with small differences (Baur and McDermontt 2010, Baur and Lucey 2010, Kaul and Sapp 2006, Upper 2000). This paper follows Baur and McDermontt (2010) for the definitions of a hedge and safe haven since their definition is most closely related to the research methodology of this paper and because Baur and McDermonnt make an important distinction between a strong/weak hedge and safe haven. For the definition of a diversifier Bouri et al. (2017) will be used.

Hedge: A strong (weak) hedge is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio on average

Safe Haven: A strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets.

Diversifier: A diversifier is an asset that has a weak positive correlation with another asset on average

At the end of Paragraph 1 is briefly mentioned that bitcoin is exposed to different factors, paragraph 2 shows by means of correlation values how this can benefit a portfolio and influence the efficient frontier. For a long time there has been a big gap in the literature about the performance of bitcoin. Only recently economist attempt to fill this gap.

To start with, where to place bitcoin as an asset? In paragraph one of this chapter is mentioned that bitcoin is a unique asset because it is a currency with characteristics of being an investment asset (Yermack, 2013; Baur, 2015). Dyhrberg (2015b) compares bitcoin to gold and the US dollar because of several similar characteristics. A comparison of bitcoin to gold and the US dollar can indicate that bitcoin could possibly be a hedge or a safe haven. Dyhrberg uses the asymmetric GARCH model to analyze the similarity between bitcoin, gold and the US dollar. The analysis shows clearly that bitcoin has medium of exchange characteristics. The results also show that bitcoin reacts roughly the same as gold, including the hedging capabilities. The only difference between bitcoin and gold is that the trading is faster and market reactions are quicker. Bitcoin is therefore classified somewhere in between a currency and a commodity. Dyhrberg concludes that bitcoin cannot only be placed between

a currency and a commodity but that bitcoin is also able to combine the advantages of a currency and a commodity. This unique combination makes bitcoin a useful tool for portfolio management and risky analysis.

Is bitcoin useful for portfolio management, does it offer diversification benefits? Eisl et al. (2015) examine if bitcoin offers any advantages on an already well-diversified portfolio. Eisl et al. make use of the Conditional Value-at-Risk framework (CVAR) instead of the classic mean-variance framework. Bitcoin has a non-normal nature, extreme volatility, as a consequence the classic mean-variance framework cannot be used since it requires returns to be normally distributed. As proxy for bitcoin the Coindesk bitcoin price index in USD is taken. The sample period runs from July 2010 (the start of the Coindesk market) until April 2015. The results show that it is optimal to add bitcoin to an already well-diversified portfolio. Even though bitcoin increases the CVAR, meaning that if the probability that the portfolio will have a loss is higher, it is optimal to invest in bitcoin because the high returns overcompensate the chance at a loss. However, the weight of bitcoin is substantial but relatively small. In the equally-weighted portfolios is straightforward that the weight is 7.69% but in the -100%/100% framework the weight varies between 1.65% till about 5%.

According to Eisl et al. (2015) does bitcoin offer diversification benefits but is bitcoin also useful for risk analysis as Dyhrberg 2015b suggest, n specific is bitcoin a hedge or a safe haven? Dyhrberg (2015a) examines the hedging capabilities of bitcoin against the Financial Times Stock Exchange index (FTSE index) and the US dollar. Dyhrberg compares bitcoin to gold and uses the wide applied asymmetric GARCH model inspired by Baur and Lucey (2010). As proxy for the bitcoin market the Coindesk Bitcoin price index is taken. The results show that bitcoin has clear overall hedging capabilities against the FTSE Index. As a hedge against the dollar the results are less clear, in the short-term bitcoin did show hedging capabilities but the correlations values were very small. Dyhrberg concludes that bitcoin is very useful for portfolio analysis and risk management as it functions as a hedge and can therefore be added to the list of instruments along-side gold for example and because of the specific speed advantages. Bouoiyour and Selmi (2017) confirm that bitcoin has hedge and safe haven properties. Bouoiyour and Selmi examine whether bitcoin can act as a hedge and safe haven for U.S stock price index and comparing it to traditional assets such as precious metals. Bitcoin's price surged in the aftermath of the U.S. elections. Bouoiyour and Selmi focus their attention therefore on the final announcement of Trump's victory on 8 November 2016 until 15 February 2017. Using the Ensemble Empirical Mode Decomposition (EEMD) proposed by Wu and Huang (2009), instead of the EMD model of Huang et al. (1998),

Bouoiyour and Selmi show that bitcoin acts as a weak safe haven in the short run and as a hedge in the medium- and the long term. At the same time, they also show that gold and silver lost their safe haven property because the co-movements of precious metals and the U.S. stock index decrease over time.

However, not all agree that bitcoin is a hedge or safe haven. Bouri et al. (2017) agrees with Eisl et al. 2015 that bitcoin offers diversification benefits but does not agree that bitcoin can also be regarded as a (weak) hedge or safe haven. Bouri et al. examines whether bitcoin can be seen as a hedge and safe haven for not only the major world stock indices but also for bonds, gold, oil, the general commodity index and the US dollar. Bouri et al. make use the dynamic conditional correlation (DCC) model of Enlge (2002). As proxy for the bitcoin market they take the Bitstamp marketplace. The data frequency differs between daily data and monthly data. The results show that bitcoin is an effective diversifier for all assets of the study and a poor hedge or safe haven. Furthermore, the hedge and safe haven effect only appears in a few cases and changes when the data frequency changes. As explanation for the difference in behavior across different horizons is referred to Ciaian et al. (2016) who state that the bitcoin price in the long run is affected by different variables as the short run price.

2.4 Hypotheses

Bitcoin the promise of the future or a doomed failure? In the first part of this chapter bitcoin's unique aspects were highlighted. Some issues raise serious concerns but others showed promising aspects. Most interesting is the finding of Ciaian et al. (2016) that bitcoin is unrelated to global macroeconomics and financial developments, alongside the findings of Yermack (2013) and Baur (2015) that bitcoin is more an investment asset than a currency. Combining these findings with portfolio optimization theory, as described in the second part, leads to the first hypotheses:

(1) Bitcoin offers diversification benefits for a global market portfolio

As Markowitz already mentioned is that for diversification purposes correlation values are very important. This is stressed by Bodie et al. (2015) be means of efficient diversification but also by Malkiel (1999) who stresses that efficient diversification is more important than the number of assets in a portfolio. Assets with negative correlations are defined as a hedge or safe haven. Recent work showed that bitcoin can act as weak hedge and safe haven in some cases (Bouri et al., 2017; Dyhrberg, 2015a; Dyhrberg 2015b; Bouoiyour and Selmi, 2017, Eisl et al. 2015). The second hypotheses and third hypotheses are therefore as followed:

(2) Bitcoin is the modern hedge

(3) Bitcoin is the modern safe haven.

3 Data & Method

The data consists of weekly data from 22 of July 2010 to 31 December 2016 retrieved from Eikon. The coindesk market is taken as proxy for the bitcoin market. The coindesk market only started trading on 17 July 2010. The first trading week therefore starts on 22 July 2010. The global market portfolio (the base portfolio) consist of the following assets:

Base portfolio		
Stock indices:	**Government bonds:**	**Others:**
S&P 500	*US bond*	*S&P GSCI commodity total*
FTSE 100	*UK bond*	*FTSE EPRA/NAREIT global*
Nikkei 225	*Japan bond*	
Shanghai A-share		
Dax 30		
MSCI World		

The stock indices are chosen to represent the most important stock markets. The MSCI world index is added to ensure the global perspective. The bonds are selected based on the three most important markets: the American market, the European market and the Asian market. Due to data access limitation no bond indices could be adopted. Including three government bonds is the next best alternative. To complete the market portfolio two important assets are also added. Namely a global commodity index and a global real estate index. For a second portfolio the dollar (in line with Dhyrberg 2015b) and gold are added to the portfolio, which is denoted as the base^{+} portfolio. As these assets are typically seen as hedges/safe havens it is interesting to see if bitcoin offers any advantage when these assets are already in the portfolio.

This paper uses the mean-variance analysis (MVA) combined with a Monte Carlo Simulations (MCS). The method of this paper combined with the applied perspective enhances the novelty of this paper and gives it two advantages. First, by using the MVA and a global investor perspective the paper has a more practical investment approach[6] instead of pure theoretical models based on correlations. Second, one of the greatest benefits of using the MVA is that this method can uniquely incorporate policy constraint. Section 2 explained the theory behind the MVA. The unique policy constraints can be found by altering the weights which are given by w_i (i = 1,2,...,N) and a budget constraint of $\sum w_i = 1$. Some policy examples:

- Limiting w_i to be > 0 implies that no short sales are permitted. Short sales are often difficult

[6] Although the approach should have the practical advantage over other methods such as the correlation models it is not widely applied in reality because MVA has several shortcomings.

to take and can be extremely risky.

- Limiting w_i to have a maximum of 0.25 implies that no more than 25% can be invested in a single asset. In practice portfolio managers are not likely to invest more than 25% in one single asset (Conover et al., 2009).
- Limiting w_i to be equally distributed implies an equally weighted portfolio.
- The budget constraint of $\sum w_i = 1$. With $\sum w_i < 1$ not all wealth has to be invested. With $\sum w_i > 1$ more than the initial wealth is invested.

The greatest shortcoming of the MVA is that one of the assumptions is that stock returns should be normally distributed or individuals having a quadratic utility function. Bitcoin's excess volatility (Molnár et al., 2015; Baur and Dimpfl, 2017) implies a non-normal distribution as Eisl et al. 2015 already mentioned. This paper will use this approach, nonetheless, because this paper believes it to provide a good benchmark. More importantly, there is no better alternative to approach the problem.

For the MVA part first a base portfolio will be constructed. Which consists of: six stock indices, 3 bonds, a commodity index and a real estate index. The perspective is that from a global investor, therefore are there absolute returns taken. Meaning that each asset is denominated in its own currency. Of each asset the total return index is used, this includes dividend and coupon payments and thus gives the total return of the asset.

Throughout the literature bitcoin is compared to gold and the US dollar. Partly because these assets typically are seen as safe havens. It would be interesting to see whether bitcoin offers any advantages when these traditional assets are also explicitly adopted in the portfolio. For gold the value of gold in US dollar is taken, for the US dollar this paper will follow Dyhrberg 2015b where the exchange rate of the dollar to the British Pound and the Euro is taken as proxy.

Of the Base portfolio the geometric returns are calculated, $R = LN\frac{Pt}{Pt-1}$. From the returns the average return over the period can be calculated. The returns are transformed to annual returns, $R_a = ((1+R)^N) - 1$. Ra stands for annual return and N is the number of weeks, 52. From the returns the correlation values and the covariance matrix can be derived. Then following the portfolio optimization process the efficient frontier can be constructed. To find out what effect bitcoin has on the portfolio the same steps previous explained will be repeated only including bitcoin. By comparing the efficient frontier it is immediately visible whether bitcoins offer advantages or not and by examining the correlation value the question whether bitcoin is a hedge or safe haven can be answered.

However, as mentioned in the introduction, the estimation risk can play an important role (Jorion, 1992). Especially in the case of bitcoin with the excess volatility. Performing a MCS will tackle this problem. In terms of a traditional statistical framework the random sample of the MVA can be drawn from a normal distribution with unknown returns and variance. The number of samples drawn from the distribution is finite. Therefore, another random sample of the distribution could have different means and covariance matrix. The new inputs would deliver other results which can be completely different. The way portfolio optimization is constructed the highest returns are weighted heavily, meaning that a small change in returns can have a large influence in the weight distribution. The simulation process will look as followed:

(1) Estimate the returns and covariance matrix of the historical data.
(2) Define N as the number of assets T as the number of weeks.
(3) Assume that that the estimations of step 1 are the true values.
(4) Assume that the model for the returns are $R_t = \mu + \varepsilon_t$. [7]
(5) Generate N simulated returns.
(6) Repeat step 5 T times.
(7) Calculate the average return of the T times simulated returns and construct the covariance matrix.
(8) Perform portfolio optimization by determining the market portfolio.[8]
(9) Repeat steps 2 to 7 one hundred times. A scatterplot of the simulated market portfolios will show which portfolio performs better when the estimation risk is taken into account.

[7] R_t represent the simulated return which consists of the historical return μ and error of the estimation risk ε. The historical return will be the same for each week only the error term will differ each weak therefore ε_t.

[8] In section 2.2 is shown that the market portfolio is the portfolio which lies on the efficient frontier and is tangent to the CML. The slope of the capital market line is $\frac{Rp-Rf}{\sigma}$, which is equal to the formula of the Sharpe ratio. The Sharpe ratio is the most well-known return-to-risk ratio (Modigliani and Modigliani, 1997) and according to Lo (2002) also the best understood ratio. Thus, by optimizing the Sharpe ratio the market portfolio can be determined.

4 Results

This chapter will present the main results of this paper. For robustness checks see chapter 5. Table 1 summarizes the weekly and annual return and standard deviation for all the assets. Bitcoin really stands out in terms of return but also in terms of risk. With an annual return of over 350% bitcoin performs more than 24 times as good as the second-best option which is real estate. Even though the risk is also much higher it is still relatively low, compared to the second riskiest investment bitcoin is 'only' 11 times more volatile. Thus far bitcoin shows interesting opportunities.

Table 1: Summary statistics

	Weekly return	*Weekly Std.Dev.*	*Annual return*	*Annual Std.Dev.*
S&P500	0,26%	1,85%	14,19%	13,36%
FTSE100	0,16%	2,01%	8,50%	14,46%
Nikkei 225	0,25%	2,88%	14,09%	20,74%
Shanghai A share	0,06%	3,19%	2,95%	22,98%
Dax 30	0,19%	2,85%	10,11%	20,56%
MSCI World	0,19%	1,96%	10,15%	14,16%
Us bond	0,07%	0,98%	3,59%	7,06%
Uk bond	0,12%	0,92%	6,31%	6,65%
Japan bond	0,06%	0,39%	2,95%	2,83%
Commodity index	-0,16%	2,71%	-8,17%	19,52%
Real estate	0,26%	2,01%	14,41%	14,53%
$-GBP	-0,07%	1,28%	-3,37%	9,21%
$-€	-0,06%	1,22%	-3,16%	8,79%
Gold	-0,01%	2,36%	-0,69%	17,04%
Bitcoin	2,94%	23,65%	350,82%	170,53%

Table 2 shows the correlation values of all the assets. All three bonds show negative correlations to each asset except gold. Gold shows negative correlations for the Nikkei 225 and the Dax 30. The dollar shows negative correlations for the bonds and for real estate. Partly confirming the results mentioned in the introduction. Bitcoin Shows very low correlation to every asset, in the case of the three bonds and gold even slightly negative correlation. Indicating that overall bitcoin is not a hedge or safe haven but a very effective diversifier.

Table 2: Correlation values

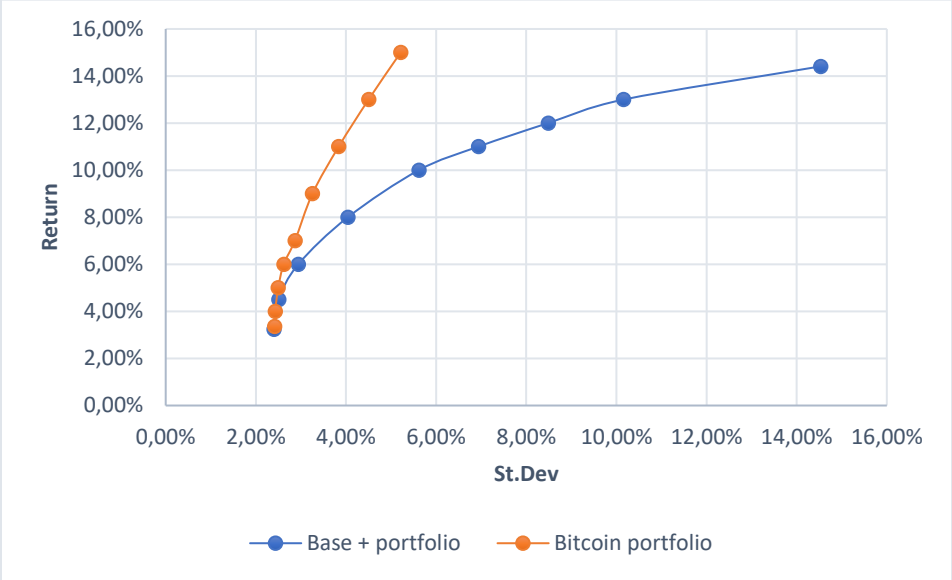
	S&P500	FTSE100	NIKKEI 225	SHANGHAI A SHARE	DAX 30	MSCI WORLD	US BOND	UK BOND	JAPAN BOND	COMMODITY INDEX	REAL ESTATE INDEX	$-GBP	$-€	GOLD	BITCOIN
S&P500	1,00														
FTSE100	0,81	1,00													
NIKKEI 225	0,95	0,56	1,00												
SHANGHAI A SHARE	0,20	0,22	0,24	1,00											
DAX 30	0,78	0,81	0,59	0,21	1,00										
MSCI WORLD	0,95	0,88	0,62	0,24	0,84	1,00									
US BOND	-0,52	-0,42	-0,37	-0,05	-0,50	-0,50	1,00								
UK BOND	-0,42	-0,34	-0,37	-0,05	-0,44	-0,45	0,81	1,00							
JAPAN BOND	-0,20	-0,21	-0,28	0,00	-0,22	-0,21	0,49	0,47	1,00						
COMMODITY INDEX	0,50	0,52	0,26	0,12	0,41	0,56	-0,34	-0,32	-0,15	1,00					
REAL ESTATE	0,60	0,61	0,38	0,13	0,54	0,57	-0,05	-0,01	-0,02	0,21	1,00				
$-GBP	0,30	0,18	0,23	0,11	0,30	0,44	-0,17	-0,34	-0,08	0,35	-0,04	1,00			
$-€	0,22	0,13	0,00	0,08	0,07	0,35	-0,09	-0,15	0,00	0,28	-0,27	0,56	1,00		
GOLD	0,06	0,05	-0,15	0,08	-0,07	0,10	0,28	0,27	0,20	0,16	0,03	0,16	0,34	1,00	
BITCOIN	0,08	0,09	0,08	0,03	0,13	0,09	-0,09	-0,09	0,00	0,03	0,04	0,01	0,00	-0,04	1,00

Graph 1 compares the efficient frontiers of the base portfolio to the base^{+} portfolio which includes the dollar and gold. The curvature of the efficient frontier is almost exactly the same. Gold is already taken into the account by the commodity index and by having the several stock indices the portfolio has a lot of assets, the theory from section 2 showed that the addition of one asset does not do much when there are already so many assets in the portfolio. As a consequence, is the efficient frontier of the base^{+} portfolio only visible higher below a return of 5%. With a return and risk of 3,24% and 2,40% the minimum variance portfolio of the base^{+} portfolio is slightly lower, compared to 3,79% and 2,50%. Since the difference in very small and in the advantage of the base^{+} portfolio this portfolio will be compared to the performance of the bitcoin portfolio. The grey line represents the capital market line and the tangency point with the efficient frontier is the market portfolio which has a slope of 1,9259 (this is also the Sharpe ratio of the market portfolio).

Graph 1: efficient frontier Base and Base + portfolio

Graph 2 compares the efficient frontier of the base^{+} portfolio to the bitcoin portfolio[9]. The result is very clear, bitcoin offers great diversification benefits. The efficient frontier is much steeper but more important also shifted to left. Meaning that for a given level of risk the bitcoin portfolio by far outperforms the base^{+} portfolio. Again, looking at table 1 and table 2 the result is not surprising. Bitcoin's return is much higher and the correlations values of bitcoin indicate that bitcoin is a very effective diversifier. The efficient frontier indeed shows that bitcoin is an effective diversifier. However as previous mentioned could the estimation risk have a large influence on the performance of both portfolios.

Graph 2: efficient frontiers



Graph 3 shows the efficient frontier of the base^{+} and the bitcoin portfolio and the simulated market portfolios. The grey dots represent the simulated market portfolios of the base^{+} portfolio and the yellow dots represent the simulated market portfolios of the bitcoin portfolio. For the results to be statistically significant the highest 5% and the lowest 5% of the simulated market portfolios are removed, leaving 90 simulated portfolios.[10]The results show that the estimation risk plays a significant role but not in the disadvantage of bitcoin. The bitcoin simulated market portfolios have a higher dispersion in returns than the base^{+} simulated market portfolios but also a higher return on average. Since the simulated market

[9] Bitcoin's efficient frontier goes further than visible in the graph but by taking a closer look at the point where the efficient frontier of the base^{+} is visible the difference becomes clearer, the complete efficient frontier can be found in the appendix.

[10] Statistically equivalent portfolios can be generated by determining a cutoff probability, 5% in this case, and discarding the lowest and highest 5% of the portfolios ranked according to the Sharpe ratio. The removal of the highest and lowest 5% of the simulated portfolios is applied to all the following analyses.

portfolio does not have to be suboptimal and as previous mentioned that the mean variance model is very senstive for small changes in return, especially for the assets with the highest return, it is not strange that most simulated market portfolios lie outside the efficiet frontier. In almost all of the bitcoin simulated market portfolios is a small percentage invested in bitcoin, see figure 8. On average 1,95% is invested in bitcoin. The results are in line with the results of the efficient frontier. The base^{+} portfolio has an average Sharpe ratio of 2,702 while the bitcoin portfolio has an average Sharpe ratio of 3,695. So far the results are all in the advantage of bitcoin being an effective diversifier and confirm the findings of Eisl et al. (2015).

Graph 3: Simulated market portfolios

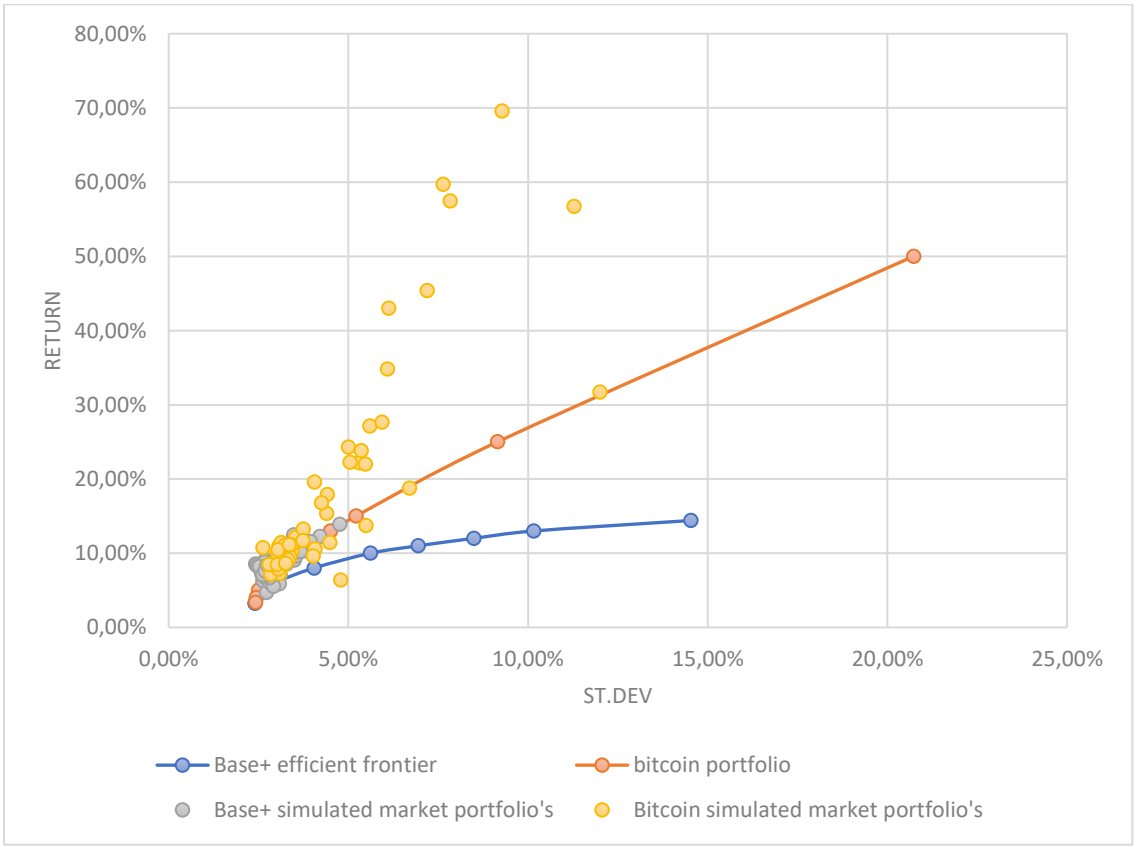
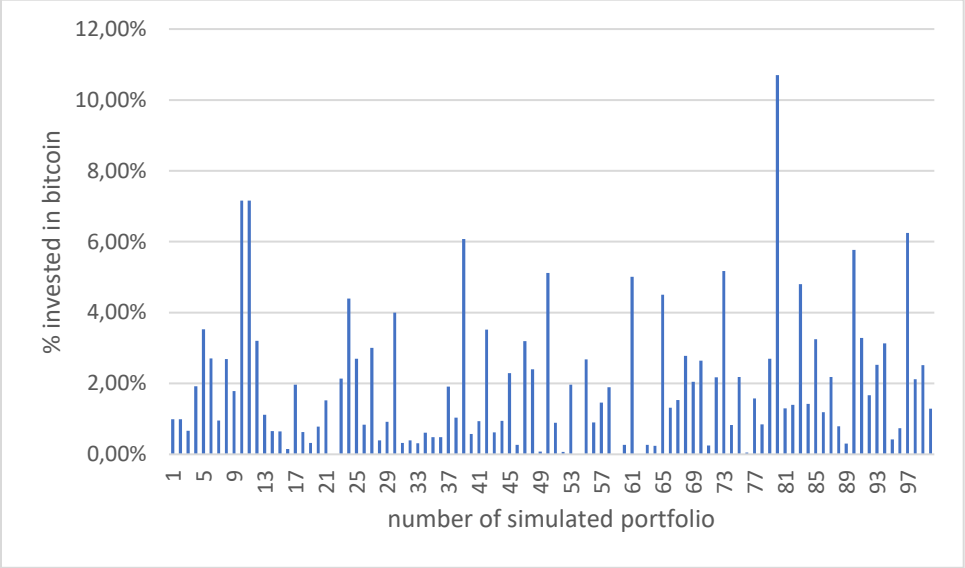


Figure 8: bitcoin's share



Different time periods

The safe haven or hedge properties bitcoin might show would indicate that bitcoin would perform better during times of uncertainty. To get a better idea on the performance of bitcoin the time period is divided into two separate periods. The periods are 2010 to 2013 and 2014 to 2016. The division is based on two main factors. One factor is that the period 2010 to 2013 can still be considered as the aftermath of the global financial crisis. In Europe for example this can partly be translated in the Sovereign debt crisis, which lasted form spring 2010 till July 2013 (Knot, 2016). The other factor is that bitcoin knows an explosive growth during November/December 2013. Filtering out this part for the 2014-2016 period will give a better indication whether bitcoin offers any advantage during 'normal' times. If bitcoin only provides advantages during the first period this would indicate that bitcoin has safe haven properties. If bitcoin also offers advantages during the second period will indicate if bitcoin shows hedge instead of safe haven properties and whether bitcoin offers advantages throughout time. Table 3 shows the correlation values during the different time period. The general performance of bitcoin is added as reference point. The complete correlation table and the summary statistics can be found in the appendix. In both periods bitcoin shows consistent low correlation with all assets. But the negative correlation against bonds become slightly more negative during the first period. Implying that bitcoin is only a safe haven for bonds. In the second period the correlations are roughly the same except that the correlations against the dollar becomes negative and the correlation against gold becomes more negative. The correlation values show that bitcoin has hedge properties for bonds and safe haven properties

for the dollar and gold. Remarkably no negative correlation values for stock can be found, which recent literature did.

Table 3: correlation values

	S&P500	*FTSE100*	*Nikkei 225*	*Shanghai A share*	*Dax 30*	*MSCI World*	*US bond*	*UK bond*	*Japan bond*	*Commodity Index*	*Real estate index*	*$-GBP*	*$-€*	*Gold*	*Bitcoin*
Bitcoin 2010-2013	0,08	0,10	0,12	0,10	0,19	0,11	-0,10	-0,11	0,02	0,02	0,06	0,10	0,05	0,05	1,00
Bitcoin 2014-2016	0,07	0,08	0,04	0,01	0,07	0,06	-0,08	-0,08	0,00	0,02	0,04	-0,05	-0,06	-0,14	1,00
Bitcoin all time	0,08	0,09	0,08	0,03	0,13	0,09	-0,09	-0,09	0,00	0,03	0,04	0,01	0,00	-0,04	1,00

Graph 5 and 6 show the performance of the simulated market portfolios during both periods. The efficient frontiers of both periods show a significant difference. Where in the whole sample period bitcoin showed a significance advantage the first period shows an even bigger advantage, this is clearly visible in graph 5. For the second period bitcoin offers still diversification benefits but the advantage is relative small. Graph 6 shows no visible difference, but there is a small positive difference for bitcoin. In the appendix a table is provided which shows the difference. About the performance of bitcoin, taking into consideration the estimation risk the bitcoin portfolio performs in most cases better than the base^{+} portfolio. Graph 5 shows that the bitcoin portfolio has a higher dispersion in both the return and the risk component but when comparing the average Sharpe ratios of all the simulated market portfolios the base^{+} portfolio has an average of 3,2599 while the bitcoin portfolio has a stunning average of 13,604. Figure 9 shows the investment behavior in bitcoin during the period. On average is 7,24% invested in bitcoin. Graph 6 is similar to graph 5 and graph 3, bitcoin shows a slightly higher dispersion in returns and risk but on average the bitcoin portfolio performs better. With an average Sharpe ratio of 3,078 the base^{+} again underperforms compared to the bitcoin portfolio which has an average Sharpe ratio of 3,313. A big difference in this period is that in not all the cases a percentage is invested in bitcoin. In 46 of the 100 simulations no investment is allocated to bitcoin. On average 0,25% is invested in bitcoin.

Overall bitcoin can be seen as a very effective diversifier. Throughout the period offers bitcoin as an investment asset diversification benefits with very low but no negative correlation values to the global market portfolio.

Graph 5: Simulated market portfolios 2010-2013

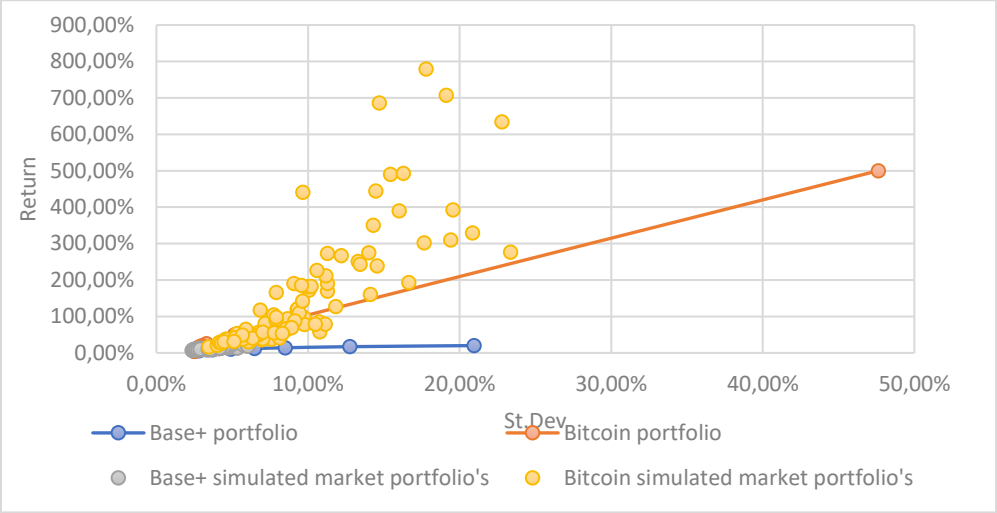
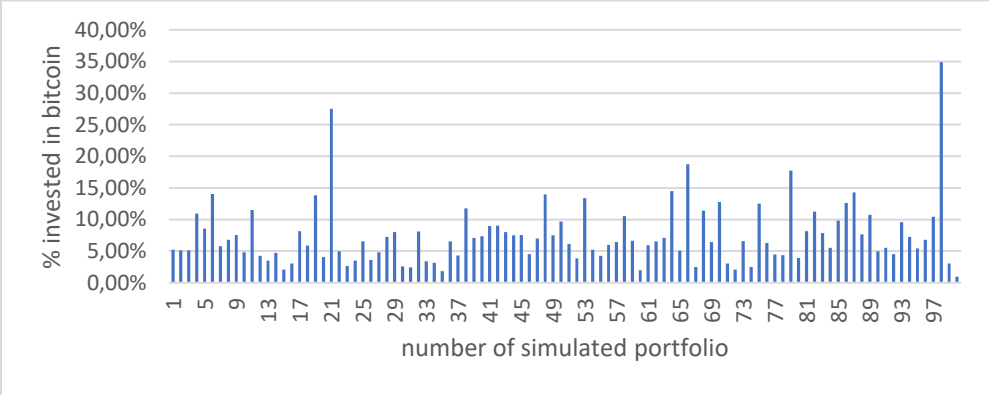


Figure 9: Allocated % in bitcoin during 2010-2013



Graph 6: simulated market portfolios 2014-2016

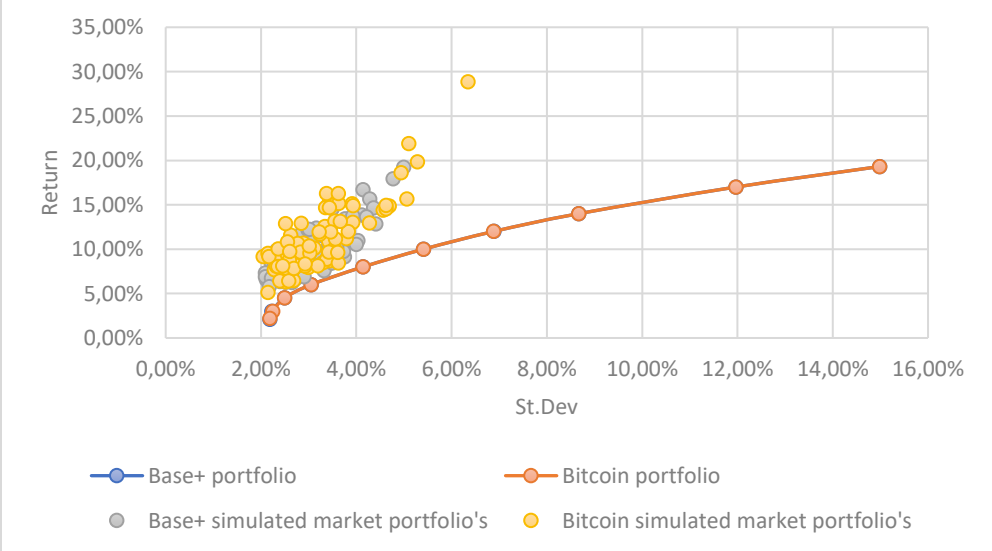
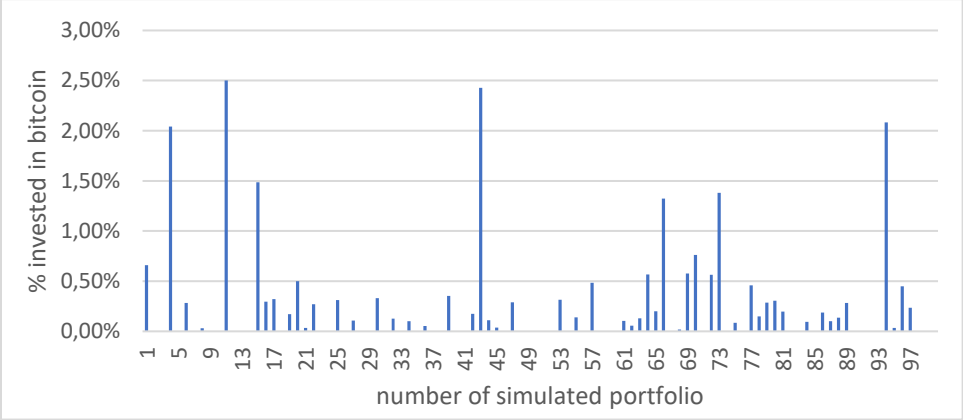


Figure 10: Allocated % in bitcoin during 2014-2016

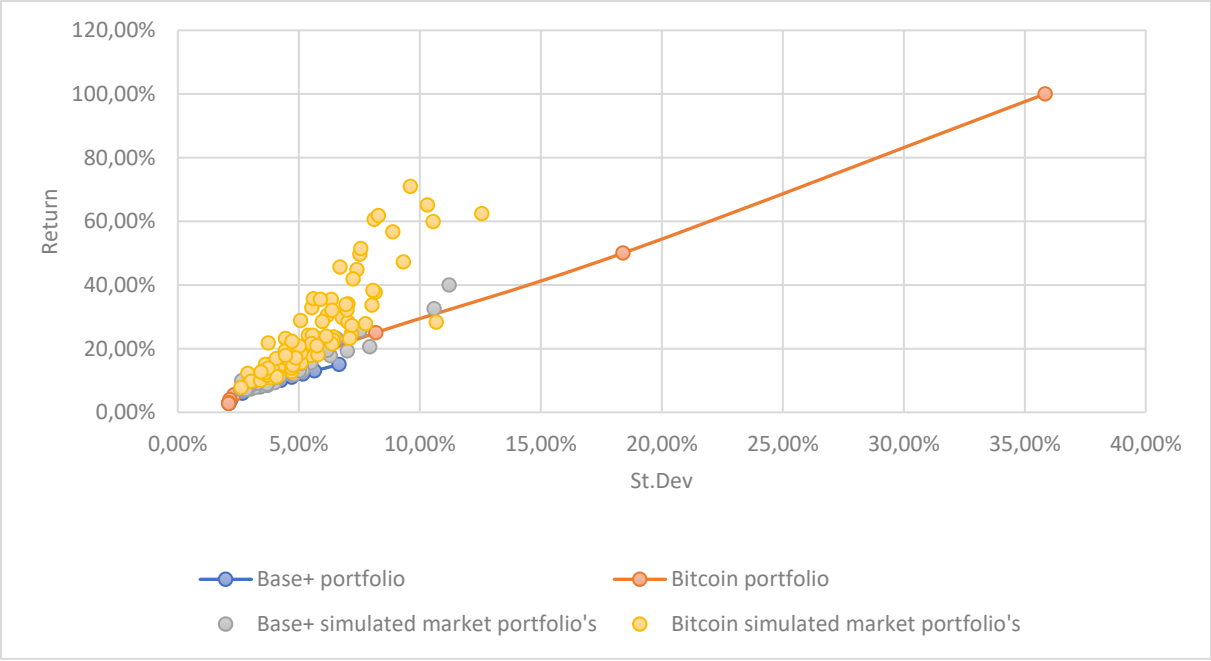


5 Robustness checks

To test whether the results of bitcoin are constant four additional analyses are performed. These four analyses each are based on different aspect to test whether certain characteristics of bitcoin have any influence on the performance of bitcoin. The analyses itself are performed the same way as the previous analyses.

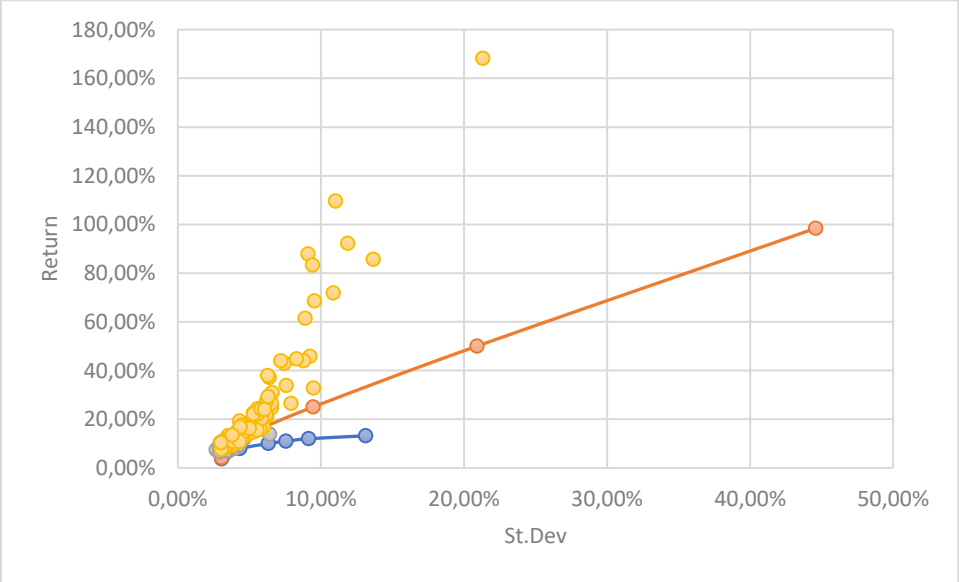
The first analysis is that the short sale constraint is removed, wi no longer has to be bigger than zero. Even though short sales are risky and can be difficult to perform in reality it can still be an important aspect of a portfolio. Due to bitcoin's large volatility it might be that bitcoin weight varies a lot even from positive to negative. Graph 7 shows the result when the short sales constraints are removed. In general, the portfolios perform better and the efficient frontiers moved to the left compared to the short sales constraint frontiers. The results are similar to the results shown before. The bitcoin portfolio has a higher dispersion in both the risk and the return component but performs on average also better. The Sharpe ratio of the base+ portfolio is 2,878, which is slightly higher than before. The Sharpe ratio of the bitcoin portfolio is 4,244, which is also slightly higher than before and higher than the base+ portfolio. On average 2,56% is allocated to bitcoin as an asset and only in one of the one hundred simulations is going short in bitcoin profitable. The investment behavior in bitcoin look similar to the previous behavior, see the appendix for the table.

Graph 7: Simulated market portfolios no short sales constraint



For the second analyses this paper follows the idea of Conover et al. 2009 that portfolio managers will not likely allocate more than 25% to one asset by adding the limitation of w_i to be smaller or equal to 0,25. Graph 8 shows the results of the simulations. Again the results are quite similar to the results previous shown. The bitcoin portfolio shows a higher dispersion in risk and return. Comparing the average Sharpe ratios the bitcoin portfolio performs better than the base$^+$ portfolio, with Sharpe ratios of 3,938 and 2,626 respectively. The base$^+$ portfolio behaves as expected, a quite good Sharpe ratio which is just slightly lower than when the constraint is removed. The bitcoin portfolio however performs slightly better, the large volatility of bitcoin can be an explanation. On average 2,35% is allocated to bitcoin furthermore looks the investment behavior similar to the no constraint behavior, see the appendix for the table.

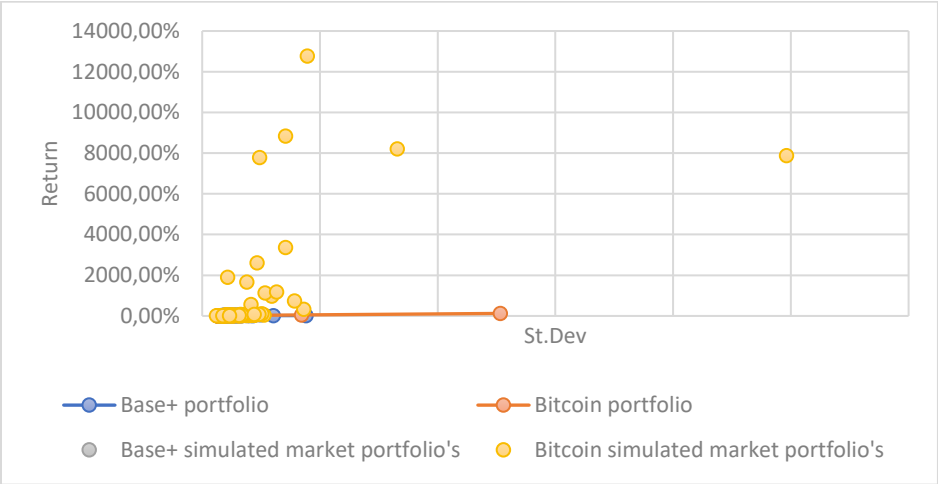
Graph 8: Simulated market portfolios with weight constraint.



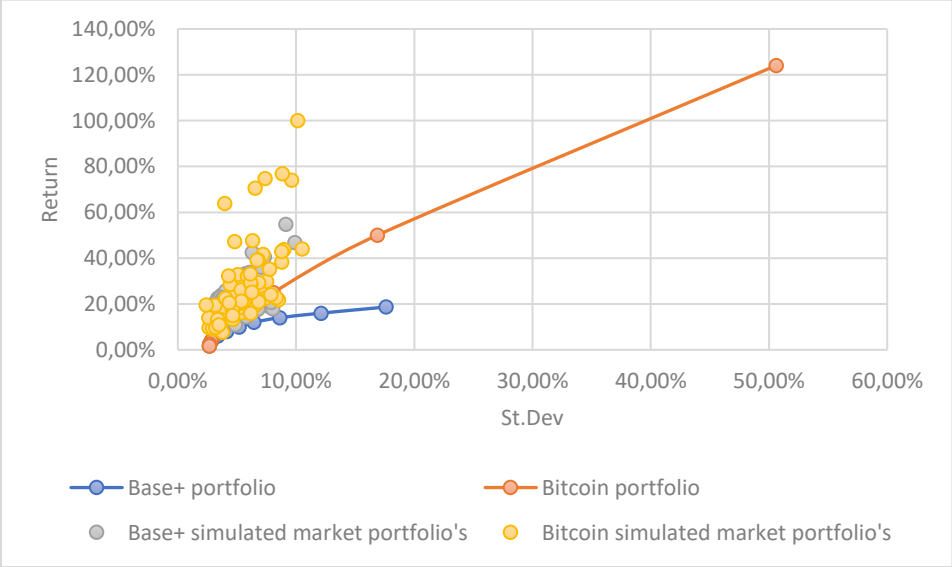
The third analysis focuses on the year 2016 because this was a year with some major events which had a large impact on financial markets. The two most important events which caused uncertainty to arise where the BREXIT and the U.S. elections. In section 2.3 this paper showed that hedges but especially safe havens would perform well in times of uncertainty. Graph 9 shows the results of the simulations. Graph 10 shows the same results but only takes a closer look at the data point which lie closer to the efficient frontier because a couple data point of the bitcoin portfolio make it very hard to read the graph correctly. Graph 9 shows that bitcoin portfolio has some simulations which have unrealistic high returns for a portfolio even with the high volatility of bitcoin. Graph 10 however shows that the bitcoin portfolio again shows a higher dispersion but also a better performance on average. The base$^+$ portfolio has

an average Sharpe ratio of 4,455, which is the highest ratio of all the different base+ portfolios. The bitcoin portfolio has an average Sharpe ratio of 49,464, also the highest ratio of all the bitcoin portfolio. The average investment in bitcoin is 0,80%, however even though the returns are high the large volatility plays an important role because in 42 of the 100 simulation is no investment made at all in bitcoin, see appendix for the table.

Graph 9: Simulated market portfolios for 2016



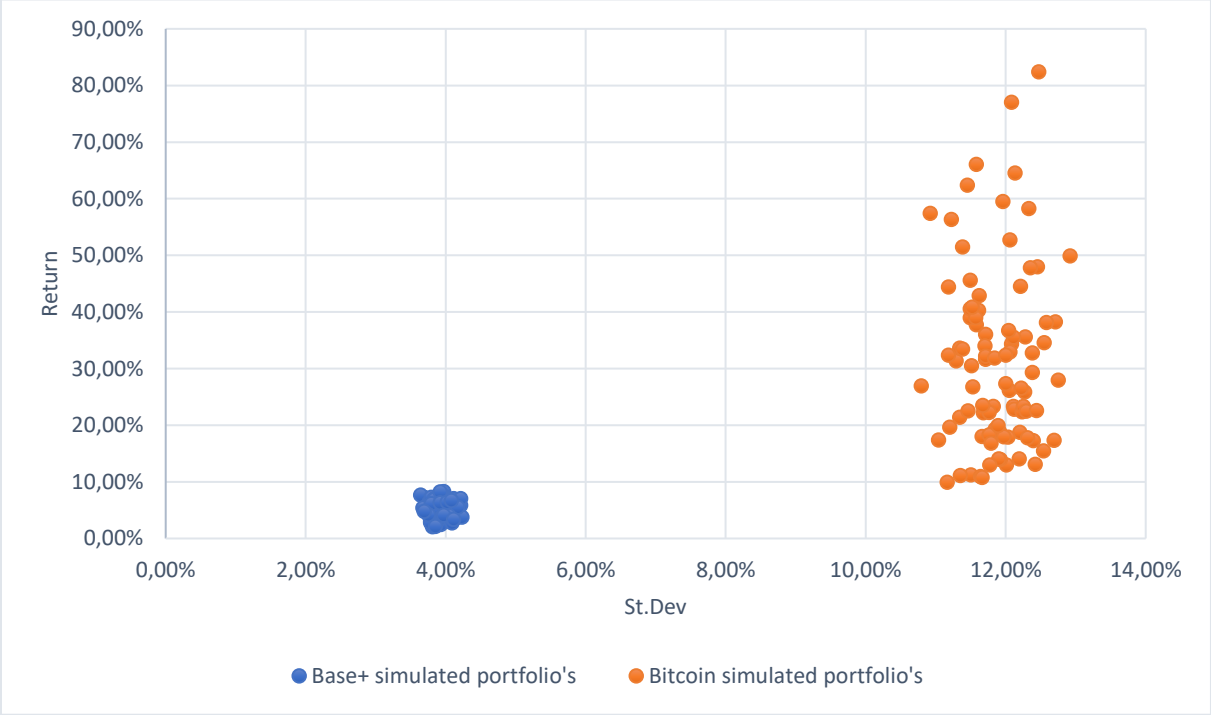
Graph 10: Close-up simulated market portfolio for 2016



The last analysis is different than all analyses before, instead of applying a portfolio optimization technique an equally-weighted portfolio is constructed where each weight is determined by 1/N (with N being the number of assets)[11]. DeMiguel et al. 2009 show than an

[11] The removal of the lowest and highest 5% also applies to this analysis.

equally-weighted portfolio performs comparable or even better in terms of Sharpe ratios. Including this analysis will only confirm whether better bitcoin truly is an effective investment and at the same time shows that there is a prediction risk involved with the risk and return measure this paper uses. Combining the prediction risk with the estimation risk the result on bitcoin should be quite robust. Graph 11 shows the results; the graph is similar to all graphs before only no efficient frontiers are added to the graph because there is no efficient frontier for an equally-weighted portfolio. In the base^{+} portfolio 7,14% is invested in each asset while in the bitcoin portfolio 6,67% is invested in each asset. Graph 11 confirms the earlier results and is in fact quite similar. The bitcoin portfolio shows a higher dispersion but has therefore also higher returns. With an average Sharpe ratio of 2,609 the bitcoin portfolio performs better than the base^{+} portfolio which has an average Sharpe ratio of 1,226. Both portfolio perform worse than all other portfolios, which is in line with the optimization theory this paper applies.



6 Conclusion

The performance of bitcoin as an investment asset in a global market portfolio stands central in this paper. Bitcoin is a decentralized digital currency which can be classified more as an investment asset than a currency. Bitcoin's growth and importance in the economy has not been gone unnoticed by the economic literature, where a growing interest around bitcoin arises. Previous literature shows that bitcoin is a very volatile asset but an asset that is also valuable for risk management. Evidence is found that bitcoin can even function as a hedge or safe haven for particular assets. From theory, a way to improve a portfolio is to find assets which have low or even negative correlations. These assets are particularly interesting in improving the shape of the efficient frontier and therefore the market portfolio. This paper applied the mean variance framework to analyze the performance of bitcoin. Since the estimation risk is seen as an important aspect due to the large volatility of bitcoin but also due to the sensitivity of the mean variance framework a Monte Carlo Simulation is performed. The general results show that bitcoin has a low correlation value and is a great asset to include in the market portfolio, confirming hypotheses one. Bitcoin's volatility does cause a higher dispersion in returns and risk but performs on average still better. When the period is divided in to two different periods the results are still in favor of bitcoin. Bitcoin showed again low correlation values and an improvement in the portfolio. There is no indication that bitcoin is hedge or safe haven which is contradictory to previous results which look at hedge and safe haven correlations, not confirming hypotheses two and three. However, when bitcoin is placed in a portfolio previous study of Eisl et al. 2015 found no hedge or safe haven properties for bitcoin, the results of this paper are thus similar and complimentary to the results of Eisl et al. 2015. This can be an explanation why adding bitcoin next to gold and the US dollar to a portfolio is beneficial. Variations on the general analyses only confirm the results. In the case that: short sales are permitted, a weight constraint with a maximum allocation of 25% to one asset or an equally-weighted portfolio is constructed the bitcoin portfolio consistently outperforms the base^{+} portfolio. The same results hold when the bitcoin portfolio is analyzed in 2016, a year with great financial uncertainty. However, bitcoin's exceptional performance in this period might indicate that bitcoin does show some signs of a safe haven asset. Generally, in all analyses is a quite low percentage allocated to bitcoin, ranging from 0% to 5%. In some cases no investment is made at all in bitcoin or an investment which exceeds the 5% level. Again the results are in line with Eisl et al. 2015 who

report an average investment around 1,65% to 5%. The following table summarizes the results of all the analyzes.

Table 4: Summary of the results

	Sharpe ratio base+ portfolio	Sharpe ratio bitcoin portfolio	% invested in bitcoin
2010-2016 portfolio	*2,67*	*3,79*	*1,95%*
2010-2013 portfolio	*3,26*	*13,60*	*7,46%*
2014-2016 portfolio	*3,08*	*3,31*	*0,25%*
No short sale constraint	*2,88*	*4,24*	*2,56%*
Weight constraint	*2,63*	*3,94*	*2,35%*
2016 portfolio	*4,46*	*49,46*	*0,80%*
Equally-weighted	*1,23*	*2,61*	*6,67%*

Even though the results of this paper show that bitcoin performs well in a global market portfolio there are some reasons for concern. As is mentioned in section 2.1 is that bitcoin is exposed to cyber-attacks, has troubles functioning as a currency and most important is that bitcoin is used in illegal activities establishing itself as the single common currency for cyber-criminals. This lead to bitcoin being banned in Thailand and recent signals from the Chinese government of a possible ban of bitcoin. On top of there is a possible issue with the performance of bitcoin. Bitcoin has a large peak at the end of 2013 and has quite strong growth in 2016. The high returns of bitcoin can possible by explained by these two points, making bitcoin on average less attractive to invest in. Both theory and the results suggest that bitcoin offers diversification benefits but is not a hedge or safe haven and an investor who holds the global market portfolio should invest in it, the reasons for concern might however indicate otherwise.

New research could focus on the general performance of bitcoin when bitcoin has matured thereby tackling the problem that bitcoin performance is probably heavily influenced by the peak at the end of 2013 and the growth in 2016. Furthermore, a possible direction is to research the hedge and safe haven properties of bitcoin in more detail from an investment perspective. Another direction could be to look whether very volatile asset such as bitcoin lead to less utility, as is briefly mentioned in section 2, and should therefore not be included in portfolios . Of course, all under the assumption that bitcoin continues to exist and not be banned by several countries or being overwhelmed by cyber-attacks or illegal activities which could kill the attractiveness to conduct research on bitcoin. However, bitcoin’s development in 2017 looks promising.

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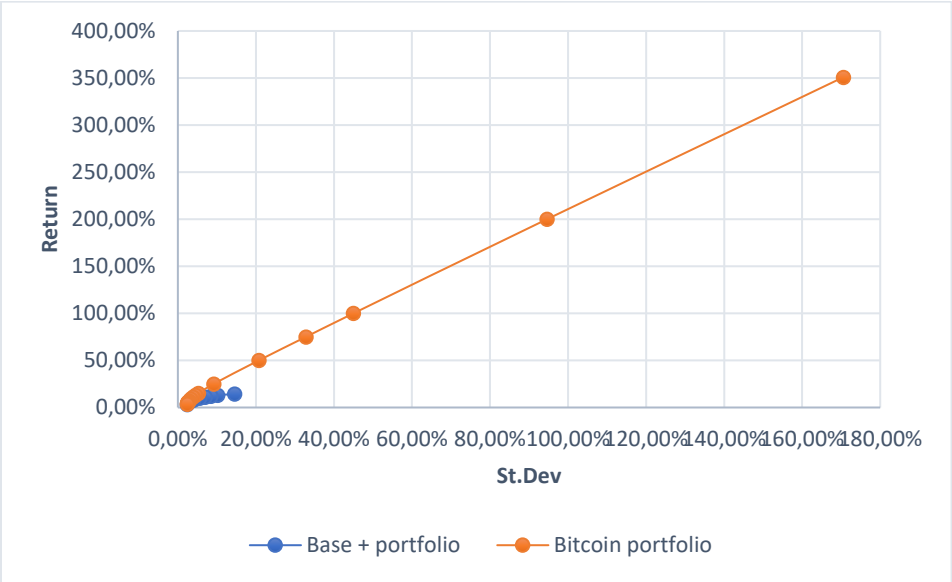
Appendix

Efficient frontiers bitcoin portfolio and base^{+} portfolio



Correlation values 2010-2013

	S&P500	FTSE100	Nikkei 225	Shanghai A s	Dax 30	MSCI World	US bond	UK bond	Japan bond	Commodity	Real estate	$-GBP	$-€	Gold	Bitcoin
S&P500	1,00														
FTSE100	0,86	1,00													
Nikkei 225	0,89	0,53	1,00												
Shanghai A share	0,23	0,26	0,20	1,00											
Dax 30	0,83	0,84	0,54	0,27	1,00										
MSCI World	0,96	0,90	0,57	0,28	0,87	1,00									
Us bond	-0,56	-0,52	-0,34	-0,11	-0,58	-0,54	1,00								
Uk bond	-0,46	-0,40	-0,36	-0,13	-0,49	-0,47	0,83	1,00							
Japan bond	-0,26	-0,27	-0,35	-0,12	-0,24	-0,24	0,50	0,49	1,00						
Commodity index	0,60	0,56	0,24	0,30	0,47	0,61	-0,37	-0,27	-0,08	1,00					
Real estate	0,66	0,64	0,39	0,09	0,57	0,61	-0,20	-0,13	-0,12	0,34	1,00				
$-GBP	0,36	0,25	0,17	0,17	0,35	0,49	-0,11	-0,22	0,07	0,37	0,04	1,00			
$-€	0,41	0,34	0,15	0,19	0,33	0,54	-0,17	-0,22	0,02	0,35	-0,11	0,66	1,00		
Gold	0,22	0,22	0,03	0,19	0,15	0,28	0,15	0,14	0,16	0,36	0,06	0,33	0,32	1,00	
Bitcoin	0,08	0,10	0,12	0,10	0,19	0,11	-0,10	-0,11	0,02	0,02	0,06	0,10	0,05	0,05	1,00

Correlation values 2014-2016

	S&P500	FTSE100	Nikkei 225	Shanghai A s	Dax 30	MSCI World	US bond	UK bond	Japan bond	Commodity	Real estate	$-GBP	$-€	Gold	Bitcoin
S&P500	1,00														
FTSE100	0,76	1,00													
Nikkei 225	0,65	0,59	1,00												
Shanghai A share	0,20	0,21	0,28	1,00											
Dax 30	0,72	0,78	0,64	0,17	1,00										
MSCI World	0,94	0,86	0,69	0,24	0,82	1,00									
Us bond	-0,44	-0,28	-0,42	-0,01	-0,40	-0,42	1,00								
Uk bond	-0,37	-0,27	-0,38	0,00	-0,39	-0,42	0,81	1,00							
Japan bond	-0,11	-0,13	-0,19	0,10	-0,19	-0,17	0,47	0,44	1,00						
Commodity index	0,40	0,48	0,29	0,01	0,35	0,51	-0,33	-0,36	-0,23	1,00					
Real estate	0,55	0,57	0,37	0,16	0,52	0,53	0,16	0,13	0,11	0,09	1,00				
$-GBP	0,25	0,12	0,29	0,09	0,27	0,41	-0,26	-0,44	-0,23	0,33	-0,09	1,00			
$-€	-0,08	-0,14	-0,20	0,01	-0,27	0,03	0,04	-0,06	-0,03	0,20	-0,47	0,50	1,00		
Gold	-0,21	-0,18	-0,41	-0,01	-0,37	-0,19	0,51	0,45	0,25	-0,07	-0,01	0,00	0,37	1,00	
Bitcoin	0,07	0,08	0,04	0,01	0,07	0,06	-0,08	-0,08	0,00	0,02	0,04	-0,05	-0,06	-0,14	1,00

Summary statictics 2010-2013

	Weekly return	*Weekly Std.Dev.*	*Annual return*	*Annual Std.Dev.*
S&P500	0,33%	2,01%	18,84%	14,46%
FTSE100	0,20%	1,99%	10,81%	14,35%
Nikkei 225	0,35%	2,91%	20,05%	20,95%
Shanghai A share	-0,12%	2,51%	-6,01%	18,08%
Dax 30	0,24%	2,83%	13,45%	20,38%
MSCI World	0,27%	2,14%	15,15%	15,40%
Us bond	0,06%	1,09%	3,23%	7,83%
Uk bond	0,09%	0,91%	4,69%	6,59%
Japan bond	0,05%	0,42%	2,74%	3,01%
Commodity index	0,08%	2,48%	4,44%	17,85%
Real estate	0,00%	1,96%	10,27%	14,10%
$-GBP	0,04%	1,04%	2,16%	7,53%
$-€	0,03%	1,28%	1,73%	9,25%
Gold	0,00%	2,55%	-0,07%	18,41%
Bitcoin	5,38%	19,06%	1427,94%	137,41%

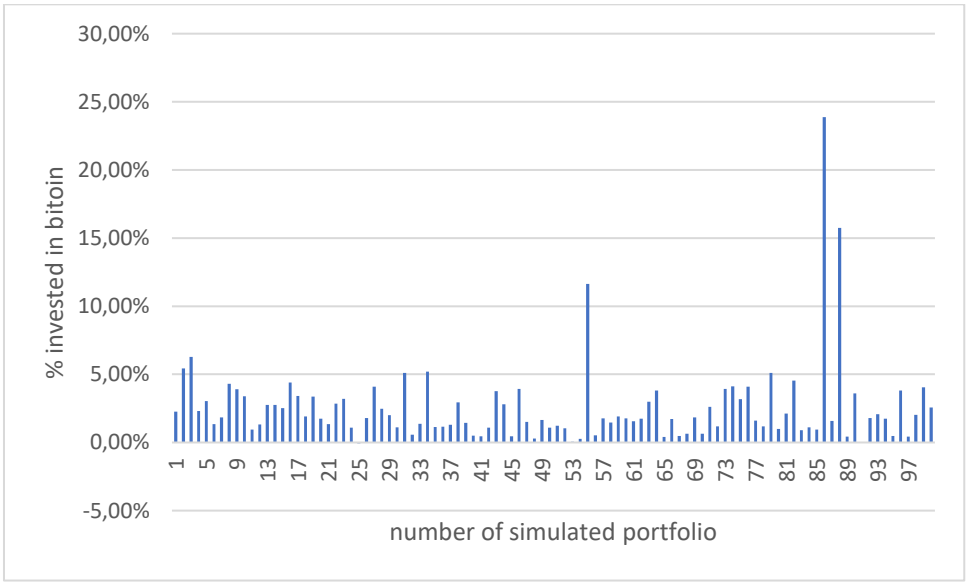
Summary statistics 2014-2016

	Weekly return	*Weekly Std.Dev.*	*Annual return*	*Annual Std.Dev.*
S&P500	0,17%	1,66%	9,11%	11,94%
FTSE100	0,11%	2,02%	5,92%	14,58%
Nikkei 225	0,14%	2,84%	7,64%	20,46%
Shanghai A	0,26%	3,81%	14,21%	27,45%
Dax 30	0,12%	2,88%	6,42%	20,76%
MSCI World	0,09%	1,74%	4,71%	12,55%
Us bond	0,08%	0,84%	4,01%	6,07%
Uk bond	0,15%	0,93%	8,19%	6,70%
Japan bond	0,06%	0,36%	3,18%	2,62%
Commodity	-0,45%	2,92%	-20,74%	21,09%
Real estate	0,34%	2,08%	19,31%	14,98%
$-GBP	-0,19%	1,49%	-9,32%	10,75%
$-€	-8,45%	1,13%	-8,45%	8,16%
Gold	-0,03%	2,13%	-1,40%	15,33%
Bitcoin	0,15%	27,72%	8,15%	199,87%

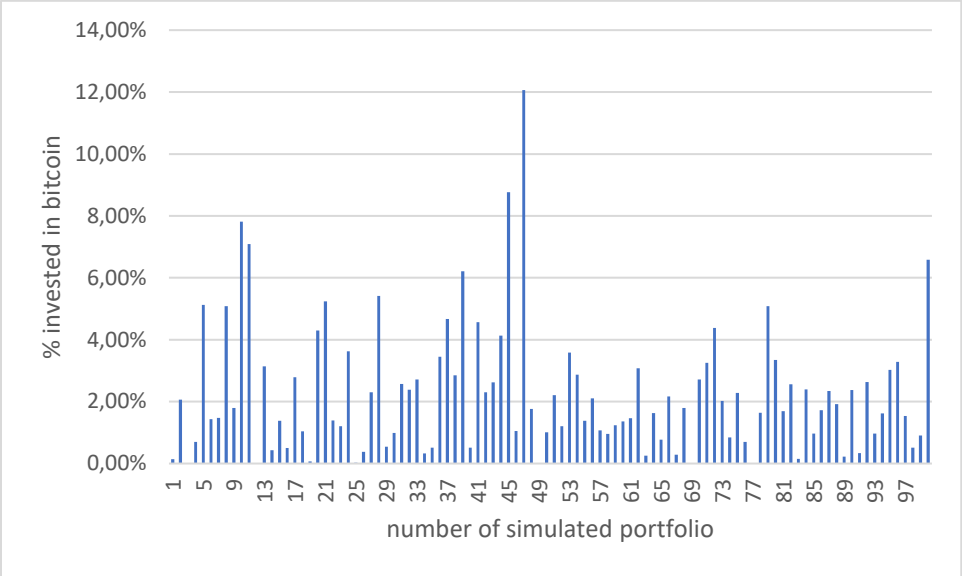
Efficient frontier 2014-2016 summary statistics

Optimal portfolio		Bitcoin portfolio	
Stdev	Returns	Stdev	Returns
14,9794%	19,3100%	14,9794%	19,3100%
11,9621%	17,0000%	11,9635%	17,0000%
8,6650%	14,0000%	8,6647%	14,0000%
6,8834%	12,0000%	6,8818%	12,0000%
5,4094%	10,0000%	5,4065%	10,0000%
4,1379%	8,0000%	4,1359%	8,0000%
3,0488%	6,0000%	3,0477%	6,0000%
2,4948%	4,5000%	2,4940%	4,5000%
2,2234%	3,0000%	2,2460%	3,0000%
2,1806%	2,0952%	2,1822%	2,1953%

Investment behavior in bitcoin with no short sales constraint.



Investment behavior in bitcoin with weight constraints.



Investment behavior in bitcoin during 2016